5. Euclid’s Geometry

1. Though Euclid defined a point, a line, and a plane, the definitions are not accepted by mathematicians. Therefore, these terms are now taken as undefined.

 2. Axioms or postulates are the assumptions which are obvious universal truths. They are not proved.

3. Theorems are statements which are proved, using definitions, axioms, previously proved statements and deductive reasoning.

4. Some of Euclid’s axioms were:

(1) Things which are equal to the same thing are equal to one another.

(2) If equals are added to equals, the wholes are equal.

(3) If equals are subtracted from equals, the remainders are equal.

(4) Things which coincide with one another are equal to one another.

(5) The whole is greater than the part.

(6) Things which are double of the same things are equal to one another.

(7) Things which are halves of the same things are equal to one another.

5. Euclid’s postulates were :

Postulate 1 : A straight line may be drawn from any one point to any other point.

Postulate 2 : A terminated line can be produced indefinitely.

Postulate 3 : A circle can be drawn with any centre and any radius.

Postulate 4 : All right angles are equal to one another.

Postulate 5 : If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

 6. Two equivalent versions of Euclid’s fifth postulate are:

 (i) ‘For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l’.

 (ii) Two distinct intersecting lines cannot be parallel to the same line.

 7. All the attempts to prove Euclid’s fifth postulate using the first 4 postulates failed. But they led to the discovery of several other geometries, called non-Euclidean geometries.

 **Questions**

1. If A, B and C are three points on a line, and B lies between A and C (see Fig. 5.7), then prove that AB + BC = AC.



2. If a point C lies between two points A and B such that AC = BC, then prove that AC = 1 2 AB. Explain by drawing the figure.

 3. In Question 2, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

4. In Fig. 5.10, if AC = BD, then prove that AB = CD.



5. Why is Axiom 5, in the list of Euclid’s axioms, considered a ‘universal truth’?